Using Lego Construction to Develop Ratio Understanding.

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This paper examines Year 7 students use and learning of ratio concepts while engaged in the technology practice of designing, constructing and evaluating simple machines, that used cogs and pulleys. It was found that most students made considerable progress in accounting for ratio concepts in their constructions and some constructed sophisticated machines and provided explicit and quantitative descriptions involving ratio reasoning. The findings have implications for the study of mathematics in integrated and contextual settings.

One of the critical questions facing mathematics education today relates to learning contexts. In particular what kinds of mathematical tools and representations are needed to promote mathematical learning and how these tools should be used (English, 2002). New technologies are giving rise to major changes in mathematics education. There are now numerous opportunities for students and teachers to engage in mathematical experiences that were scarcely contemplated a decade ago. However, the effective use of new technologies neither happens automatically, nor will the use of technology lead to improvements in mathematics learning without changes to the curriculum (Niss, 1999). Equally important is what pedagogical changes become associated with learning, with new learning contexts and tools (Kaput & Roschelle, 1999).

In terms of changes in curriculum there has been a shift in the content of mathematics, the most important shift in the last two decades has been towards increased emphasis on powerful ideas associated with mathematical processes (Jones, Langrall, Thorton, & Nisbet, 2002). NCTM Standards (2004) has encapsulated this trend world wide by giving pre-eminence to five process standards: problem solving, reasoning and proof, connections, communication and representation. This shift in curriculum approach towards communication of reasoning and integration, or contextual problem based learning reasoning has found expression in attempts to integrate science and technology with mathematics. For example, the New Basics curriculum documents (Education Queensland, 2001) has a strong emphasis on integrated learning. This document encourages teachers to use integrated community based activities, where the role of the teacher is one of mentoring while students engage in tasks that are relevant and authentic to the students. Other curriculum documents have recommended an approach to mathematics teaching and learning that is integrated or transdisciplinary and were the mathematics is embedded in authentic contexts (e.g., National Council of Teachers of Mathematics, 2004; Queensland Studies Authority, 2003). Such tasks tend to enable students to develop modelling capacities that need greater mathematizing and the conceptual use of mathematics (e.g., Nason & Woodruf, 2003).

In the middle school years of mathematics, many topics require ratio reasoning skills. It is required for example, in applications of percentages, rate, ratio, the study of trigonometry and proportion applications. Lamon (1995, p. 169) defines ratio “as a comparative index that conveys the abstract notion of relative magnitude.” The difficulties in teaching ratio are further illustrated by the very small fraction of early adolescents, who can apply numerical approaches meaningfully in addition, the critical importance of ratio (and proportional understanding) has been noted previously (e.g., Karplus, Pulos, & Stage, 1983). Resnick and Singe (1993) put forward the hypothesis that early abilities to reason non numerically about the relations among amounts of physical material, provide the child with a set of relational schema that eventually apply to numerically quantified material, and later to numbers as mathematical objects. In the process of teaching ratio it has been recommended students be given time to explore and discuss authentic ratio and proportional situations/problems and not to be placed in the situation where algorithmisation and automatisation clogs the process of insight development (e.g., Ben-Chaim, Fey, Fitzgerald, Benedetto, & Miller 1998). Thus the purpose of this paper is to explore the learning of ratio when students design, make and appraise artefacts in which ratio thinking is embedded.

Approach and Methodology

This paper reports on students learning of ratio concepts within the context of a larger study involving the integration of mathematics, science and technology syllabus outcomes. The research approach was one of participatory collaborative action research (Kemmis & McTaggart, 2000). The researcher established a working relationship with the teachers and taught most of the 2 hour lessons over a 10 week period. The collection of data included observations of students' interactions with objects, peers and teachers, students planned and constructed artefacts, their explanations of how things worked, and written tests.

Subjects

The subjects were 56 Year 7 students in two classes in a State primary school in Brisbane. The school was a trial school for New Basics Curriculum (Education Queensland, 2001) that attempts to integrate the teaching of subject domains of science and mathematics into authentic project based tasks. The two classroom teachers were also part of the study. Jill (all names are pseudonyms) was a very experienced primary school teacher who also had extensive tertiary teaching experience and a passion for science. The second teacher, Cameron was also experienced in teaching this year level. In addition he had completed a science degree.

Procedure and Instruments

For most lessons the students worked on constructions in groups of two or three and the researcher moved between groups facilitating construction of artefacts and encouraging them to explain their artefacts in terms of science and mathematics principles. In some lessons, 10 to 15 minutes was spent in whole group discussion of the underlying theories.

Prior to beginning teaching the researcher and Jill matched science outcomes (Queensland School Curriculum Council, 1999) and mathematics outcomes (Queensland Studies Authority, 2003) with construction activities related to the "Simple and Powered Mechanisms" kits (Lego Educational Division, 2003) that were designed to help students learn engineering concepts. The kits contains a motor, various cogs and pulleys, various blocks, axles, connecting pieces as well as instruction booklets. In the first half of the intervention, students constructed artefacts with the assistance of the Lego plans (Lego Educational Division, 2003). Thereafter they increasingly constructed from their own plans.

The principal mathematics concepts were; velocity, revolutions, linear measurement, circumference (with radius, and pi), quantification of gear ratios and quantification of pulley mechanisms (ratio and proportional reasoning). Combined with the technology and science outcomes, the intervention covered a considerable range learning outcomes. This report focuses on student development of ratio.

Prior to the implementation of the study, the students were pre-tested for knowledge related to ratio and proportional reasoning - mathematics concepts associated with the planned activities. The pencil and paper tests had 11 questions that examined ratio. Some questions were simply definitions, for example: "A small car motored across the room. What is this motion called and what units were used to measure it?" (rate). Further questions required quantitative expressions of proportional reasoning to gain full marks for- example:

> Examine the diagram of pulleys below. If the circumference of pulley A is 20cm, the circumference of pulley B is 40cm and the circumference of pulley C is 10cm and pulley B I spun twice, describe how pulleys A and C will spin. Explain why this will occur.

Other questions were more open, for example, students were given the number of teeth on various gear cogs and asked to use their knowledge of bicycles to create a matching pair that would help the bike go fast, and to give a possible explanation that would be correct and complete. Scoring was on the basis of correctness and completeness of explanations, and each item was allocated 2 marks. Answers that had correct quantitative responses as well as their explanations were allocated full marks. In the second week of the study the students also planned and constructed an artefact that either served a practical need or modelled a useful product. At the end of the study students repeated the planning and construction activity, and the science and mathematics tests. Throughout the study the teachers acted as observers and documented student activity that indicated that outcome had been meet.

Analysis

The written pre and post test data were converted to percentages. The pre and post tests are compared using simple paired *t* tests. Student artefacts were examined for the application of engineering principles such as leverage and gear ratio that reflected ratio concepts. In assessing the artefacts and assessing the associated explanations the descriptions of ratio cited above (Lamon,1995) was used. Thus, student explanations and constructions were examined with respect to the correctness and completeness in terms of the application and articulation of ratio principles. Throughout the study students were asked a number of times to explain their artefacts. A hermeneutic cycle (Guba & Lincoln, 1994) was employed in developing and testing assertions as the study progressed. Emerging assertions were discussed with the teachers and colleagues and tested and refined in the light of further evidence. Triangulation involved the use of multiple data sources identified above and this maximised the probability that emergent assertions were consistent with a variety of data.

Results

The results are presented as a number of assertions.

Assertion One: Most students improved in their ability to explain science and mathematics concepts on pencil and paper tests.

Descriptive statistics and mean comparisons are contained in Table 1 below. Descriptive statistics indicate that the assumption of homogeneity of variance was not violated. All tests had approximately normal distributions of scores. There was a significant difference in the mean scores for science as indicated by the paired *t* test result, [$t(55)=10.26$, $p < 0.000$}, with the students scoring higher on the post-test. A number of students performed poorly in both pre- and post-tests. A few students made hardly any progress at all, supporting earlier findings that ratio is a concept that some students find very difficult to grasp (e.g., Resnick & Singer, 1993).

Table 1
Pre and Post-Test Paired Results on Ratio Questions as Percentage of Total.

Test	N	mean	SD
Maths pre-test	56	33.04	17.59
Maths post-test	56	60.59**	21.37

**Significance level at $p < 0.01$

Some students gave correct quantitative answers as well as correct qualitative explanations. For example students were shown a picture of two meshing cogs- a small one *B* and a larger one *A,* and asked to explain the effect of turning *A* twice on the smaller gear *B*. Some students counted the teeth of the cogs (13 on *B* and 24 on *A*) and responded "*A* turns 1.846 times while *B* turns once." Most students responded with qualitative answers such as "*A* turns about twice for each turn of *B*." While these responses take account of the ratio, a few students gave responses that took account to proportional reasoning to suggest "*B* will turn about 4 times to make *A* turn twice."

The data indicate that most students' ability to answer pencil and paper tests based on ratio concepts improved over the course of the study. Still, it is somewhat disappointing that despite some 16 hours of engagement on construction activities associated with ratio the mean final score was only about 60%. On the other hand, students are not expected to demonstrate an understanding of ratio concepts until Level 5 (Queensland Studies Authority, 2003) which equates to junior secondary school but is often introduced in Year 7. Thus, students who were able to demonstrate an understanding of ratio were operating in advance of what was expected of them. Further, ratio was not the only focus concepts, because as noted earlier, science and technology outcomes were also being developed.

Assertion Two: Most students improved in their ability to use and explain ratio and proportional reasoning in their construction of artefacts.

Selected students described in this discussion are exemplars of those who made representative gains in ability to use and explain science and mathematics principles associated with their artefacts. All groups made considerable improvement in the quality of the plans and constructions they created. Most notable was the use of gearing to effect an outcome, improved application and explanation of ratio and a better application and explanation of the mathematical principles inherent in circles and understanding of velocity. Perhaps the best example is of the pair of girls (Sarah and Mary), who designed and constructed the car shown in Figure 1 in week two. Sarah explained that this car was designed for all terrain travel. It featured the use of a simple pulley system to drive a four wheel drive "tractor" using a 40 tooth cog gear for rear wheels to "help with grip." The design illustrates that the students did not understand the relationship between wheel diameter and circumference. When they tested the car, it would run on a desk where slippage negated the lack of synchronization between the front and back wheels, but would not run on carpet where the front wheels acted as a brake on the back wheels. Despite prompting, the girls could not diagnose the problem without explicit scaffolding that directed them to consider how far each wheel would turn for each revolution of the motor. In this instance, not only did the students not recognise the significance of the relationship between the different diameters and the distance each wheel would travel, they assumed the ratio of front and rear wheel travel was equivalent. Interestingly, the students had very recently studied the relationship between diameter and circumference, but were unable to use this knowledge in a novel context, that is, they had instrumental rather than relational understandings (Skemp, 1987) of this concept.

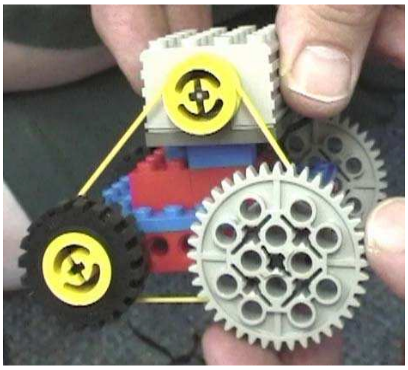

Figure 1. Prototype tractor.

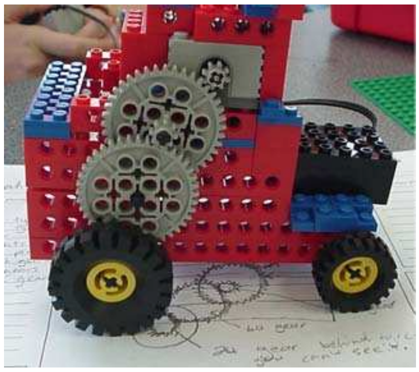

Figure 2. Final tractor, gearing ratio 15:1.

In contrast, in week 8 Sarah and Mary had designed the tractor illustrated in Figure 2. This tractor won a tug of war competition against other student constructed tractors. Sarah and Mary could now both explain the use of multiple cogs to create an overall gearing ratio of 15:1. Their diagrammatic explanation of the gearing is seen on the planning sheet below the tractor in Figure 2. Their written explanation included:

> The 8 teeth gear on the motor (driver) will turn the 40 teeth gear underneath it, (the driver) will turn 5 times, then 8 gear on the same bar as the 40 teeth gear will also turn 5 times, when the 8 gear turns 5 times, the gear with 40 teeth will turn once, making the 24 teeth gear that is behind the wheel turn around about 1 and ¾ times.

The students were aware that the drive had to turn 25 times to effect an outcome of "about 1 and ¾ times." The girls had initially tried to use a 40 tooth cog on the final drive but used a 24 tooth cog on this model. They used a 24 tooth cog to solve a construction difficulty (cog matching). Had the girls not used the 24 teeth cog the final ratio would have been 25:1. Clearly, the girls have presented very strong evidence that they had progressed in their ability to use ratio, and their description of the final ratio-25 is to "about 1 and ¾," is remarkably accurate. These girls also made considerable advances in their pre and post-tests on ratio and proportion related questions (Sarah, 43% to 77%; Mary, 58 to 84%). Consistent with the test data, not all students made such gains in their ability to apply and explain ratio concepts in the construction and explanation of their artefacts. For example, the boys who constructed the racer in Figure 3 below were able to give a qualitative explanation for their gearing:

> Well the big pulley will go round once, but it will make the driver pulley go round lots and make the car go really fast.

This explanation demonstrated a qualitative understanding between the diameter and the circumference of each pulley and the relative circumferences of each pulley.

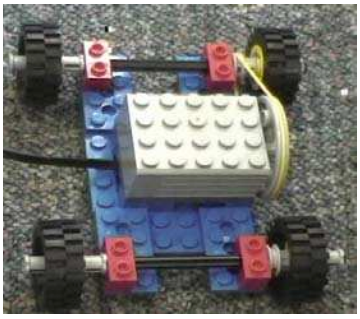

Figure 3: Pulley racer.

Even students who made very limited gains in the written test produced products that at least accounted for 1:1 drive ratios, although they did not make these relationships explicit. For example, in describing the workings of a 4 wheel drive with all pulleys of 1cm diameter, the following description was offered:

> The motor is connected to the wheels with a rubber band, when the motor makes the first wheels turn, the others turn and that makes it work.

This statement makes no attempt to describe the relationships that exist between the pulley sizes nor how this effects their functioning. None the less, the car was functional.

Assertion Three: Most students made gains in their ratio and proportional reasoning abilities over the life of the study and this was directly related to the technology activities.

Jill commented favourably on the activities as a means of teaching science and mathematics, for example she stated:

> I do believe that they have a far better idea of ratio. I think that the activities really cemented ratio. The practical application with the gearing that was really good, because they had a visual as well a practical application and it helped them to put it all together. It was a great grounding and is going to stand by them for year eight and beyond, they will always recall this.

Further she noted lack of student motivation in the traditional way mathematics was taught to this class:

> They were very de-motivated in terms of maths. They hated it, because they felt that they were not good at it. That is why I have adopted a thematic approach.

Jill commented on student motivation:

> By and large, with the exception of probably about four people in the class, I believe that each child valued it. Clearly to me learning took place. They loved playing with it, you know, the actual building of it. I think there was a sense of commitment there, the commitment to keep working for so many weeks.

Clearly, Jill considered that student motivation during the construction activities was a factor that contributed to their mathematics learning. However, she recommended increasing the connections between the activities and outside experiences of the children such as "thrill rides and roller coasters and computer programs and about how thing work." The importance of linking various representations of the ratio concept has been noted in research literature (e.g., Ben-Chaim et al., 1998; Lamon, 1995). Jill further recommended that links between construction activities and formal mathematics be made.

> I would keep them in the same sort of structure because this group needs structure, structure but let the kids explore and talk as well. You need to make links to board work, to set time limits, have set

class discussion. They need a bit more time to absorb the information and build the artefacts because some are thinking, "am I doing this right?"

The second teacher, Cameron, supported Jill's evaluation of students learning and like Jill he recommended that the links between the activities and formal mathematics be made more explicit:

> It is definitely more hands on (than a normal mathematics lesson) and appeals to a student who likes to see things in their hands and count gears and so on. But, maybe some students didn't see the structure (underlying mathematics concept)…For that to happen maybe some needed more direction, a bit more board work. Do the ratio with board work and then tell them, "OK lets apply that knowledge to building with this Lego now."

Discussion

It has been previously noted that the traditional way that ratio has been taught, lead to a lack of transferability of that knowledge (e.g., Ben Chaim et al. 1998). This study demonstrates that use of concrete materials such as Lego offer a mathematically rich environment where the powerful idea of ratio is used by students in problem solving and reasoning contexts that have personal meaning to them as has been recommended (e.g., Education Queensland, 2001; Jones et al. 2002; NCTM, 2004). The student results on pencil and paper tests, the student constructions and explanations indicated that the activities were rich in opportunities to promote the learning of ratio. The teachers' comments suggested that the learning gains were mostly due to student involvement in the intervention, rather than some other factor such as maturation or alternative learning activities. Given the difficulty level of these concepts, (Karplus et al., 1998; Resnick & Singer, 1993) such an improvement in results supports the suggestions of those authors who recommend a multi representational of mathematical concepts, including the use of manipulative material as an approach to teaching ratio (e.g., Ben-Chaim, et al. 2002; Lamon, 1995). The results also encourage the adoption of an integrated approach to teaching powerful ideas, as reflected in reform curriculum documents (e.g., Education Queensland, 2001) and situated learning contexts (Brown, Collins & Duguid, 1989). It also supports the suggestion that the use of Lego is a very useful manipulative for the learning of mathematical ideas. A number of authors (e.g., McRobbie, Norton & Ginns, 2003) expressed a concern that the powerful ideas associated with such activities, including Lego construction and robotics could remain latent. Both classroom teachers in this study recognised this as a potential issue for some students. The comments of the teachers indicated that they believed that at least, to some degree, the learning of ratio could have been better, with more explicit linking of the mathematical ideas and the construction activities. This was especially recommended for students who struggled to make the links between the construction tasks and the abstraction of mathematical ideas. The number of students who did not make ratio a part of their description of their artefacts supports this suggestion. This study adds to our capital of pedagogical knowledge about how the use of such tools can foster learning. In particular, it indicates that a clear focus on specific mathematics outcomes may be necessary. It is recommended that teachers undertake professional development that includes learning to identify specific mathematics outcomes inherent in construction activities, and ways of using Lego materials in the classroom.

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